

## Mock JEE Advanced-4 (CBT) | Paper – 2 | JEE 2024

Date: 15/05/2024

Maximum Marks: 180

Timing: 2:30 PM to 5:30 PM

Duration: 3.0 Hours

### General Instructions

- The question paper consists of 3 Subject (Subject I: **Physics**, Subject II: **Chemistry**, Subject III: **Mathematics**). Each Part has **four** sections (Section 1, Section 2, Section 3 and Section 4).
- Section 1** contains **4 Multiple Choice Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.  
  
**Section 2** contains **3 Multiple Correct Answers Type Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE THAN ONE CHOICE** is correct.  
  
**Section 3** contains **6 Non-Negative Integer Type Questions**. The answer to each question is a **NON-NEGATIVE INTEGER**. For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.  
  
**Section 4** contains **2 Paragraphs**. Based on each paragraph, there are **TWO (02)** questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places
- For answering a question, an ANSWER SHEET (OMR SHEET) is provided separately. Please fill your **Test Code, Roll No.** and **Group** properly in the space given in the ANSWER SHEET.

Name of the Candidate (In CAPITALS) : .....

Roll Number : .....

OMR Bar Code Number : .....

Candidate's Signature : ..... Invigilator's Signature .....

**MARKING SCHEME**

**SECTION – 1 | (Maximum Marks: 12)**

- This section contains **Four (04)** Multiple Choice Questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme.  
*Full Marks* : +3 If **ONLY** the correct option is chosen.  
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered).  
*Negative Marks* : –1 In all other cases.

**SECTION – 2 | (Maximum Marks: 12)**

- This section consists of **Three (03)** Questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:  
**Full Marks:** +4 If only (all) the correct option(s) is(are) chosen  
**Partial Marks:** +3 If all the four options are correct but **ONLY** three options are chosen  
**Partial Marks:** +2 If three or more options are correct but **ONLY** two options are chosen and both of which are correct  
**Partial Marks:** +1 If two or more options are correct but **ONLY** one option is chosen, and it is a correct option  
**Zero Mark:** 0 if none of the options is chosen (i.e. the question is unanswered)  
**Negative Marks:** –2 In all other cases.

**SECTION – 3 | (Maximum Marks: 24)**

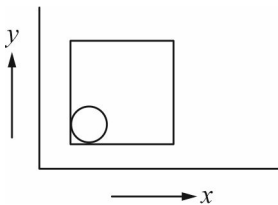
- This section contains **SIX (06)** Questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme.  
*Full Marks* : +4 If **ONLY** the correct integer is entered;  
*Zero Marks* : 0 In all other cases.

**SECTION – 4 | (Maximum Marks: 12)**

- This section contains **Two (02)** Paragraphs. Based on each paragraph, there are **TWO (02)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme.  
*Full Marks* : +3 If **ONLY** the correct numerical value is entered in the designated place.  
*Zero Marks* : 0 In all other cases

**SUBJECT I : PHYSICS****60 MARKS****SECTION-1**

This section consists of 4 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

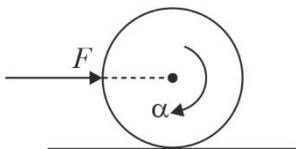
- Find the potential difference  $V_{AB}$  between  $A(2m, 1m, 0)$  and  $B(0, 2m, 4m)$  in an electric field,  
 $E = (x\hat{i} - 2y\hat{j} + z\hat{k})V/m$   
 (A) 2 Volt (B) 3 Volt (C) 5 Volt (D) 7.5 Volt
- Speed of light  $V$  is expressed in terms of three derived quantities, namely, angular momentum  $L$ , frequency  $F$  and intensity (Power/area)  $S$ , as  $v = L^a F^b S^c$ , then value of a, b, c respectively are:  
 (A)  $2, \frac{1}{2}, \frac{-1}{2}$  (B)  $\frac{1}{2}, \frac{-1}{1}, 2$  (C)  $\frac{1}{2}, 2, \frac{1}{2}$  (D)  $\frac{1}{2}, 2, \frac{-1}{2}$
- A sphere of mass 1 kg rests at one corner of a cube. The cube is moved with a velocity  $v = (8t\hat{i} - 2t^2\hat{j})$ , where  $t$  is time in second. The force by sphere on the cube at  $t = 1s$  is ( $g = 10ms^{-2}$ ) [Figure shows vertical plane of the cube]  
  
 (A) 8 N (B) 10 N (C) 20 N (D) 6 N
- If nitrogen gas molecule goes straight up with its rms speed at  $0^\circ C$  from the surface of the earth and there are no collisions with other molecules, then it will rise to an approximate height of:  
 (A) 8 km (B) 12 km (C) 20 km (D) 15 km

**SECTION-2**

This section consists of 3 Multiple Correct Answers Type Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE THAN ONE CHOICE** is correct.

- A beam of light of wavelength 600nm from a distant source falls on a single slit 1.0 mm wide and the resulting diffraction pattern is observed on a screen 2m away. Which of the following statements are correct?  
 (A) The distance between the first dark fringe on either side of the central bright fringe is 2.4 mm.  
 (B) The distance between the first dark fringe on either side of the central bright fringe is 1.2 mm.  
 (C) The angular divergence is  $12 \times 10^4$  rad  
 (D) The angular divergence is  $6 \times 10^4$  rad

6. Consider a uniform disc of mass  $m$ , radius  $r$ , rolling without slipping on a rough surface with linear acceleration ' $a$ ' and angular acceleration  $\alpha$  due to an external force  $F$  as shown in the figure. Coefficient of friction is  $\mu$ .



For the above situation, which of the following statement(s) is/are correct?

- (A) Work done by friction force at instant of pure rolling is zero.
- (B) Magnitude of friction force in case of pure rolling is  $\frac{ma}{2}$ .
- (C) Angular momentum is conserved, in case of pure rolling, about a point at height  $\frac{3r}{2}$  above surface.
- (D) Angular momentum about instantaneous axis of rotation varies linearly with time.
7. The electric field associated with an electromagnetic wave propagating in a di-electric medium is given by  $\vec{E} = 15(3\hat{i} - 4\hat{j}) \sin[2\pi(10^{15}t - 10^7z)] V/meter$ . Which of the following option(s) is/are correct?

[Given: The speed of light in vacuum,  $C_0 = 3 \times 10^8 m/s$ ]

- (A)  $B_x = 6 \times 10^{-7} \sin[2\pi(10^{15}t - 10^7z)] Wb/m^2$
- (B)  $B_y = 4.5 \times 10^{-7} \sin[2\pi(10^{15}t - 10^7z)] Wb/m^2$
- (C) The wave is polarized in xy-plane with polarization angle  $53^\circ$  with respect to x-axis.
- (D) Refractive index of medium is 3.

### SECTION-3

This section consists of 6 NON-NEGATIVE INTEGER Type Questions. The answer to each question is a **NON-NEGATIVE INTEGER**. For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.

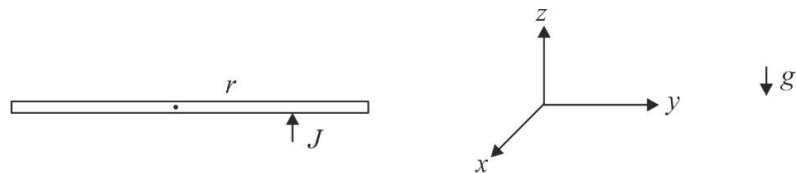
1. A uniform rod of mass  $m$ , length  $l$  is given impulse  $J$  in vertical direction at distance  $r$  from middle of rod. The rod spins and moves upwards due to this. By the time rod comes back to its initial position, it completes  $n$  rotations. The value of  $n$  is \_\_\_\_\_. Given that,

(i)  $J = \sqrt{\frac{\pi}{2}} \times 10^{-2} N - \text{sec}$

(ii)  $l = 10 \text{ cm}$

(iii)  $r = 4 \text{ cm}$

(iv)  $m = \sqrt{240} \text{ grams}$

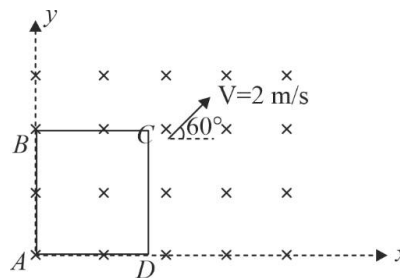


2. A square loop ABCD of side length  $= a$  is moving along  $xy$  plane with constant velocity

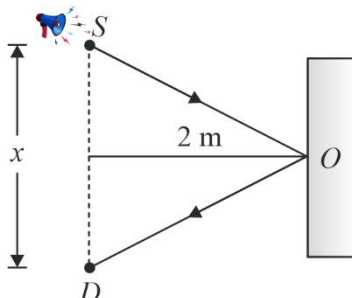
$\vec{v} = (\hat{i} + \sqrt{3}\hat{j}) \text{ m/sec}$ , in the space, where magnetic field  $\vec{B}$  is given by,

$\vec{B} = -B_0 \frac{x}{L} \hat{k}$ , where  $B_0$  and  $L$  are numerical constants. If resistance of loop ABCD is  $10\Omega$  and given that

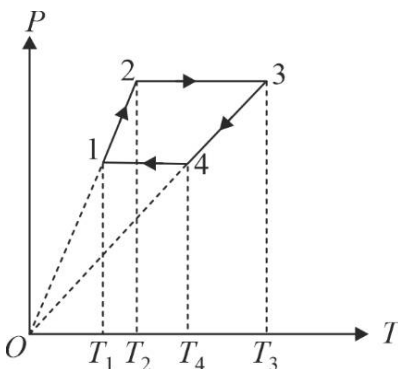
$\frac{B_0 a^2}{L}$  equals  $2 \times 10^{-5}$ . Find the induced current (in  $\mu A$ ) when corner A just crosses origin.



3. A source of sound emitting waves at 360 Hz is placed in front of a vertical wall, at a distance 2 m from it. A detector is also placed in front of the wall at the same distance from it. Find the minimum distance (in centimetre) between the source and the detector for which the detector detects a maximum of sound. Take speed of sound in air = 360 m/s. Assume that there is no phase change in reflected wave.



4. A solid sphere of diameter 0.1 m is at  $427^\circ\text{C}$  and is kept in an hollow enclosure at  $27^\circ\text{C}$ . Take Stefan's constant  $= \frac{20}{3} \times 10^{-8} \text{ W/m}^2 \text{K}^4$ , emissivity of the surface 0.84, specific heat 0.1 kcal/kg K, density = 9280  $\text{kg/m}^3$ ,  $J = 4200 \text{ J/K cal}$ . If rate of decrease of temperature of the sphere is  $N \times 10^{-3} ^\circ\text{C/s}$ , find  $\frac{N}{40}$ .
5. In a radio-active decay process, the activity is defined as  $A = -\frac{dN}{dt}$ , where  $N(t)$  is the number of radio-active nuclei at time  $t$ . Two radio-active sources,  $S_1$  and  $S_2$  have activity  $A_{10}$  and  $A_{20}$  at  $t = 0$ , whose ratio is 4. At some later time, activities of  $S_1$  and  $S_2$  are  $A_1$  and  $A_2$  are. When  $S_1$  and  $S_2$  have just completed their 4<sup>th</sup> and 7<sup>th</sup> half-lives, respectively, the ratio of  $\frac{A_1}{A_2}$  is \_\_\_\_\_.
6. n moles of an ideal gas is taken through a four step cyclic process as shown in the diagram. Work done by the gas in a cycle in terms of temperature  $T_1, T_2, T_3$  and  $T_4$  is  $mnRT_1$ , where m is a non-negative integer, given that  $T_1 : T_2 : T_3 : T_4$  equals (1:2:6:3), then m is \_\_\_\_\_.



### SECTION-4

This section consists of 2 Paragraphs. Based on each paragraph, there are **TWO (02)** questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places

#### Paragraph For Question 7 & 8

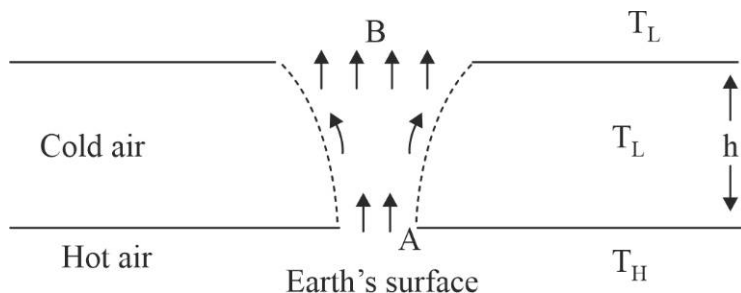
A man of mass 50 kg starts running on a plank of mass 150 kg with speed of 8 m/s relative to plank as shown in the figure. Plank is placed on smooth horizontal surface. The man while running, whistles with frequency  $f_0$ . A detector (D) placed on plank detects frequency. The man jumps off with same velocity from point D and slides on the smooth horizontal surface [Assume coefficient of friction between man and surface is zero]. The speed of sound in still medium is 330 m/s. Answer the following questions on the basis of above situations. ( $f_0 = 9072\text{Hz}$ )



7. The frequency of sound detected by detector D, before man jumps off the plank is \_\_\_\_\_. Hz.
8. The frequency of sound detected by detector d, after man jumps off the plank is \_\_\_\_\_ Hz.

#### Paragraph For Question 9 & 10

When earth's surface becomes very hot (temperature =  $T_H = 320\text{K}$ ), and due to abrupt weather change, over the layer of hot air, a thick layer of cold air (temperature =  $T_L = 273\text{K}$ ) is formed, the hot air starts rising up forming a funnel. Density of hot air at bottom equals  $1.3\text{ kg/m}^3$  and molar mass of air equals 30 grams per mole (gas constant R equals  $8.3\text{ JK}^{-1}\text{mol}^{-1}$ ). For the given non-rotating drift of hot air, solve the following question, taking air to be acting as ideal gas, and gas is undergoing iso-thermal changes. ( $g = 10\text{ m/s}^2$ )

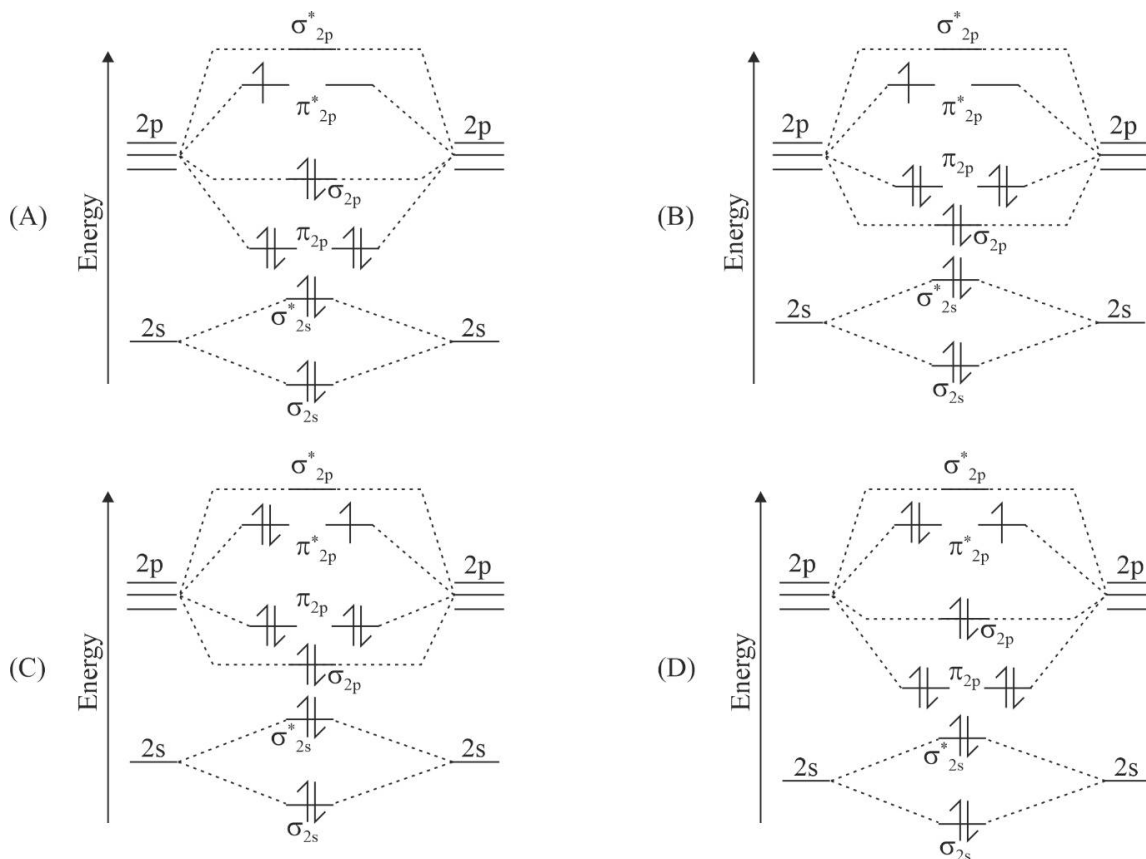


9. Find suction velocity (in m/s) at bottom of funnel (i.e. point A) if delivery rate at top is negligible in comparison, given that height of funnel, h equals 1 km.
10. Find density of cold air (in  $\text{kg/m}^3$ )

**SUBJECT II : CHEMISTRY****60 MARKS****SECTION-1**

This section consists of 4 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

1. The correct molecular orbital diagram for  $O_2^+$  molecule in the ground state is:



2. Consider the following statements related to colloids.

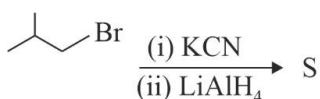
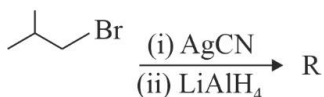
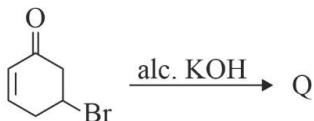
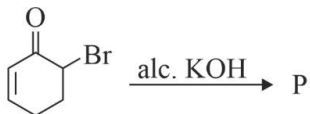
- (I) Within a typical cell, proteins and nucleic acids are colloidal sized particles dispersed in aqueous solution of ions and small molecules.
- (II) Whipped cream is a foam, which is a liquid dispersed in a gas.
- (III) The range of diameters for colloids particles is in between 1 and 1000 nm.
- (IV) Lyophilic sols are also named as irreversible sols.

The option with the correct set of statements is:

- (A) (I) and (II)      (B) (II) and (III)      (C) (I) and (III)      (D) (II) and (IV)

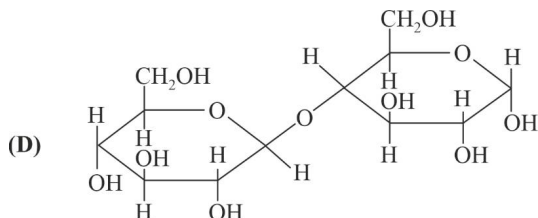
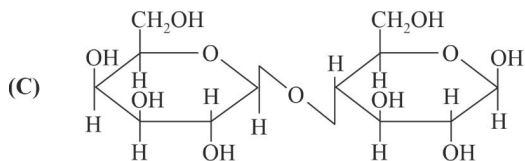
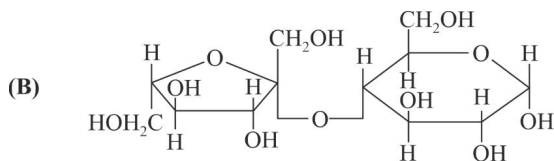
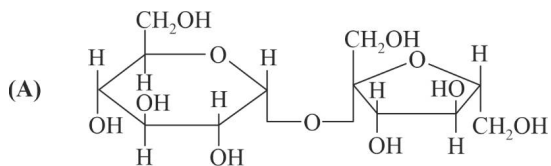


3. In the following reactions P, Q, R and S are the major products.



The incorrect statement about P, Q, R and S is:

- (A) R is secondary amine with five carbons.  
 (B) P and Q are non identical  
 (C) When chlorobenzene is fused with NaOH at 623K and high pressure followed by acidification produces P.  
 (D) R and S are functional isomers.
4. A disaccharide 'P' does not give Tollen's test. Treatment of 'P' with copper (II) ions complexed with tartrate ions do not give any brick red precipitate. The disaccharide 'P' is.



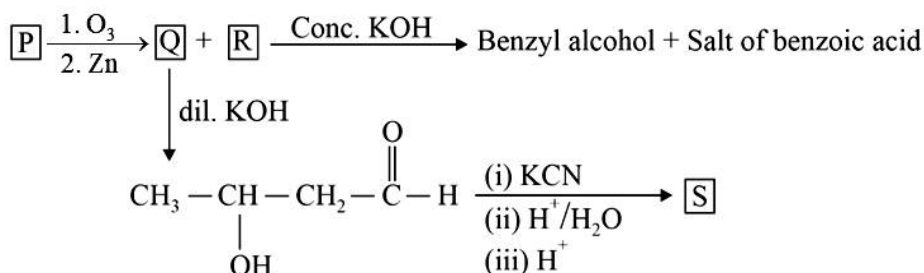
## SECTION-2

This section consists of 3 Multiple Correct Answers Type Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE THAN ONE CHOICE** is correct.

5. The complex(es), which can exhibit the type of magnetic behaviour shown by  $K_3[Cu(CN)_4]$

- (A)  $K_4[Fe(CN)_6]$  (B)  $[Cu(NH_3)_4](NO_3)_2$   
 (C)  $[Fe(H_2O)_4NO]SO_4$  (D)  $Na_2[Fe(CN)_5NO]$

6. For the following reaction sequence:



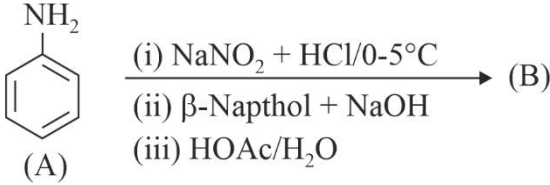
The correct statement(s) about P, Q, R and S is (are)

- (A) Structure of P is  $\text{Ph} - \text{CH} = \text{CH} - \text{CH}_3$   
 (B) Structure of Q and R differentiated by Tollen's test.  
 (C) Structure of Q and R differentiated by Fehling's test  
 (D) Compound S is a cyclic ester.
7. In diamond structure, carbon atoms produces fcc lattice and 50% tetrahedral voids held by carbon atoms. Each carbon atoms is neighboured tetrahedrally by four carbon atoms with bond length 154 pm. ( $N_A = 6 \times 10^{23} \text{ mol}^{-1}$ )
- (A) The mass of diamond unit cell is 96 amu.  
 (B) The side length of diamond unit cell is 355.65 pm  
 (C) The density of diamond unit cell is  $35.6 \text{ gm/cm}^3$   
 (D) The coordination number of corner carbon atom is 8.

**SECTION-3**

This section consists of 6 NON-NEGATIVE INTEGER Type Questions. The answer to each question is a **NON-NEGATIVE INTEGER**. For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.

- Tollen's test helps in differentiating Aldehydes from ketones. Tollen's reagent being a mild oxidising agent, oxidises aldehydes but not able to oxidise ketones. During this test, the overall number of electrons transferred to the Tollen's reagent  $[\text{Ag}(\text{NH}_3)_2]^+$  to produce silver mirror from one aldehyde group is \_\_\_\_\_.
- In how many of the following species, the central atoms have two lone pairs of electrons?  

$\text{XeF}_4$	$\text{XeF}_5^-$	$\text{F}_2\text{SeO}_2$
$\text{XeF}_3^+$	$\text{XeOF}_4$	$\text{ClOF}_3$
$\text{ICl}_4^-$	$\text{SCl}_2$	$\text{OSF}_4$
$\text{ClF}_3$	$\text{PCl}_5$	$\text{BrF}_5$
- The sum of oxidation states of iron atoms in the given compounds is x.  
 $[\text{Fe}_2(\text{CO})_9]$ ,  $[\text{Fe}(\text{NH}_3)_6]\text{SO}_4$ ,  $\text{Na}[\text{Fe}(\text{NH}_3)_2(\text{CN})_4]$   
 $\text{K}_3[\text{Fe}(\text{CN})_6]$ ,  $\text{Na}_4[\text{Fe}(\text{CN})_5(\text{NOS})]$   
 $\text{Na}_4[\text{FeO}_4]$   
 The value of x is \_\_\_\_\_.
- Electrons in a sample of H-atoms undergo transitions from state  $n = x$  to some lower excited state. The emission spectrum from the sample contains only the lines belonging to a particular series. If one of the photons had an energy of 0.6375 eV. Then find the value of x. [Take  $0.6375 \text{ eV} = \frac{3}{4} \times 0.85 \text{ eV}$ ]
- 0.2 g of Arsenic is dissolved in 17.2 g of benzene ( $\text{C}_6\text{H}_6$ ) lowers down the freezing point from  $6.49^\circ\text{C}$  to  $6.30^\circ\text{C}$ . If  $K_f$  of benzene is  $4.9 \frac{^\circ\text{C}}{\text{m}}$ , the atomicity of the molecule is:  
 (At. Wt. of As = 75)
- Consider the following reaction sequence from A to B as shown below  


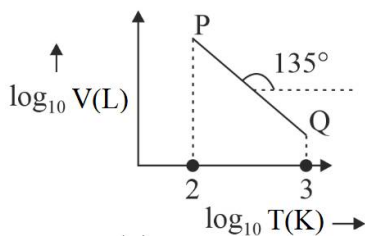
If 15.5 mL of A undergoes reaction to produce B. The overall yield for the formation of B as the major product is 50%. The mass of B obtained is \_\_\_\_\_. [Nearest integer]  
 [Density of A =  $1.20 \text{ g mL}^{-1}$ , molar mass of C = 12.0, H = 1.0, O = 16.0 and N = 14.0  $\text{g mol}^{-1}$ ] (Nearest Integer)

**SECTION-4**

**This section consists of 2 Paragraphs.** Based on each paragraph, there are **TWO (02)** questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places

**Paragraph For Question 7 – 8**

$\log_{10} V$  vs  $\log_{10} T$  plot for three moles of an ideal gas undergoing a reversible change in state from P to Q is given.



7. Heat supplied to the gas is  $x \times R$ . Value of  $x$  is \_\_\_\_\_.

( $R$  : Gas constant, Take  $C_V = \frac{5R}{2}$ )

8. Work done on the gas is  $y \times R$ . value of  $y$  is \_\_\_\_\_.

( $R$  : Gas constant, Take  $C_V = \frac{5R}{2}$ )

**Paragraph For Question 9 – 10**

A Nitro compound 4-Nitro aniline, on complete reaction with excess bromine produces a major product, which on further treatment with  $\text{NaNO}_2 / \text{HCl}$  at  $0-5^\circ\text{C}$  provides a product 'P'. 'P', upon treatment with excess  $\text{CuBr}/\text{HBr}$  gives a major product, which on reduction with excess of  $\text{Sn}/\text{HCl}$  gives a product 'Q'. The compound 'Q' upon treatment with  $\text{NaNO}_2 / \text{HCl}$  at  $0-5^\circ\text{C}$  followed by treatment with Hypophosphorous ( $\text{H}_3\text{PO}_2$ ) acid in aqueous medium gives 'R'.

9. The molar mass of product 'R' is \_\_\_\_\_

[Use: Molar mass (in  $\text{gm mol}^{-1}$ ) :  $\text{H} = 1$ ,  $\text{C} = 12$ ,  $\text{N} = 14$ ,  $\text{O} = 16$ ,  $\text{Br} = 80$ ,  $\text{Cl} = 35.5$ ]

10. The total number of carbon atoms and heteroatoms present in one molecule of Q is \_\_\_\_.

**SUBJECT III : MATHEMATICS****60 MARKS****SECTION-1**

This section consists of 4 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

- Each of 2010 boxes in a line contains one red marble and for  $1 \leq k \leq 2010$ , the box at the  $k^{\text{th}}$  position also contain  $k$  white marbles. A child begins at the first box and successively drawn a single marble at random from each box in order. He stops when he first draws a red marble. Let  $p(n)$  be the probability that he stops after drawing exactly  $n$  marbles. The smallest possible value of  $n$  for which  $p(n) < \frac{1}{2010}$  is:  
 (A) 44 (B) 45 (C) 46 (D) 47
- If the solution of the differential equation  $e^{\frac{x}{y}(1-y^2)} \left[ y \frac{dx}{dy} - x \right] + \left[ xy^2 + y^3 \frac{dx}{dy} \right] = 0$  is:  
 $Ae^{xy} + Be^{\frac{x}{y}} + C = 0$ , Then  $A + B$  is:  
 (A) -1 (B) 0 (C) 2 (D) 1
- Consider a function  $f(x) = \sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-x^2}{1+x^2} + \tan^{-1} \frac{2x}{1-x^2} - a \tan^{-1} x$  ( $a \in R$ ), then sum of all possible values of 'a' if  $f(x) = 0 \forall x \in R - \{0,1\}$  is:  
 (A) 8 (B) -6 (C) 2 (D) -2
- Given that  $\vec{a}$  and  $\vec{b}$  are two unit vectors such that angle between  $\vec{a}$  and  $\vec{b}$ , is  $\cos^{-1} \left( \frac{1}{4} \right)$ . If  $\vec{c}$  be a vector in the plane of  $\vec{a}$  and  $\vec{b}$ , such that  $|\vec{c}| = 4, \vec{c} \times \vec{b} = 2\vec{a} \times \vec{b}$  and  $\vec{c} = \lambda \vec{a} + \mu \vec{b}$  then,  
 (A)  $\lambda = 1$   
 (B) sum of values of  $\mu = 1$   
 (C) product of all possible values of  $\mu = -12$   
 (D) number of distinct values of  $\mu$  is three.

**SECTION-2**

This section consists of 3 Multiple Correct Answers Type Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE THAN ONE CHOICE** is correct.

5. A differentiable function  $f : R \rightarrow R$  satisfies the functional equation

$f(x) \cdot f(y) + f(x+y) = e^x f(y) + e^y f(x) + xy \forall x, y \in R$ . If  $f'(0) = 0$  and  $f(0) = 0$ , then which of the following statements is/are correct?

(A)  $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = \frac{1}{2}$

(B)  $\int_x^{x^2} (f'(t) - f(t)) dt > 0 \forall |x| > 1$

(C)  $F(x_2) > F(x_1) \forall x_2 > x_1$ , where  $F(x) = f'(x) - f(x)$

(D) There exist at least two horizontal tangents to the curve  $y = f(x)$  in  $(-1, 1)$ ,

6. Let  $A, B, C$  be  $n \times n$  real matrices and are pair wise commutative and  $ABC = O_n$  and if

$\lambda = \det(A^3 + B^3 + C^3) \cdot \det(A + B + C)$  then

(A)  $\lambda > 0$

(B)  $\lambda < 0$

(C)  $\lambda = 0$

(D)  $\lambda \in (-\infty, \infty) - \{0\}$

7. Let  $g(x) = ax^2 + bx + c$ ,  $a, b, c \in N$  and  $\int_0^1 g(x) dx = \frac{11}{6}$ . Let  $f(x)$  be a continuous and derivable

function in  $(x_1, x_2)$ . If  $f(x) \cdot f'(x) \geq x \sqrt{1 - (f(x))^4}$  and  $\lim_{x \rightarrow x_1^+} (f(x))^2 = 1$  and  $\lim_{x \rightarrow x_2^+} (f(x))^2 = \frac{1}{2}$ , then

minimum value of  $\left[ x_1^2 - x_2^2 \right]$  is equal to:

Where  $[.]$  denotes greatest integer function.

(A)  $2 - abc$

(B)  $a + b - c$

(C)  $b - c$

(D)  $a - b$

### SECTION-3

This section consists of 6 NON-NEGATIVE INTEGER Type Questions. The answer to each question is a NON-NEGATIVE INTEGER. For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.

- Each of the nine balls can be placed with equal probability in any of the three initially empty boxes. Let  $P_1$  be the probability that there will be four balls in the first box, three in the second box and two in the third box, and  $P_2$  be the probability that three will be in each box then the value of  $72 \cdot \frac{P_2}{P_1}$  is equal to \_\_\_\_\_.
- A  $3 \times 3$  determinant has entries either 1 or  $-1$ . Let  $S$  be the set of all determinants such that the product of elements of any row or column is  $-1$ . For example  $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix}$  is an element of  $S$  and number of elements in  $S$  is  $m$ . Let  $P = \begin{bmatrix} 3 & -2 & 3 \\ 2 & -2 & 3 \\ 0 & -1 & 1 \end{bmatrix}$  and trace of the matrix  $\text{adj}(\text{adj } P)$  is  $n$  then the value of  $\frac{m}{n}$  is:
  - Let  $f: R \rightarrow R$  be a function defined by  $f(x) = x^3 - 3x^2 - 9x + 27$ . If  $x \in (a, b) \cup (c, d)$  then  $f^3(x) - 3f^2(x) - 9f(x) + 27 < f(x^3 - 4x^2 - 3x + 19)$ . Then  $(b - a) + (c + d)$  is:  $(a < b < c < d)$
  - Let  $f$  be the real valued differentiable function on  $R$ . such that  $e^{-x} f(x) = \frac{3}{e^2} + 4e^{-x} \int_2^x \sqrt{2t^2 + 6t + 5} dt$   
 $\forall x \in R$  and let  $g(x) = f^{-1}(x)$  then  $[g'(3)] + [g''(3)]$  is equal to \_\_\_\_\_ (where  $[.]$  denote the greatest integer function).
  - Three circles  $C_1, C_2, C_3$  with radii  $r_1, r_2, r_3$  ( $r_1 < r_2 < r_3$ ) respectively are given as  $r_1 = 2$ , and  $r_3 = 8$  they are placed such that  $C_2$  lies to the right of  $C_1$  and touches it externally,  $C_3$  lies to the right of  $C_2$  and touches it externally. There exist two straight lines each of which is a direct common tangent simultaneously to all the three circles then  $r_2$  is equal to:
  - Let  $z_1, z_2, z_3$  be complex numbers (not all real) such that  $|z_1| = |z_2| = |z_3| = 1$  and  $2(z_1 + z_2 + z_3) - 3z_1z_2z_3$  is real. Then,  $\text{Max}(\arg(z_1) + \arg(z_2) + \arg(z_3))$   
 (Given that argument of  $z_1, z_2, z_3$  is positive) has minimum value as  $\frac{k\pi}{6}$  where  $(k + 2)$  is \_\_\_\_\_.

### SECTION-4

**This section consists of 2 Paragraphs.** Based on each paragraph, there are **TWO (02)** questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places

#### Paragraph "I"

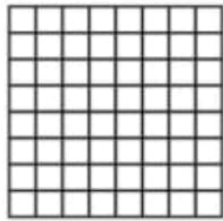
Incircle of  $\triangle ABC$  touches  $AB, BC, CA$  at  $R, P$  and  $Q$  respectively. If  $\frac{2}{AR} + \frac{5}{BP} + \frac{5}{CQ} = \frac{6}{r}$ , (where  $r$  = in radius of the incircle) and the perimeter of the triangle is the smallest integer then answer the following questions:

**(There are two questions based on PARAGRAPH "I", the question given below is one of them)**

7. The inradius of the  $\triangle ABC$  is \_\_\_\_\_.
8. The area of the  $\triangle ABC$  is \_\_\_\_\_.

#### Paragraph "II"

Consider the  $8 \times 8$  square in the figure.



Let  $A_1, A_2, \dots, A_{81}$  be the points of intersections of the lines in above figure in some order. We say that  $A_i$  and  $A_j$  are friends if they are adjacent along a row or along a column. Assume that each of these 81 points have an equal chance of being chosen.

**(There are two questions based on PARAGRAPH "II", the question given below is one of them)**

9. Let  $P_i$  denote the probability that a randomly chosen point has ' $i$ ' friends.  $i = 0, 1, 2, 3, 4$ .  
Let  $X$  denote the random variable such that for  $i = 0, 1, 2, 3, 4$ , the probability that  $P(X = i) = P_i$  Then value of  $36E(X)$  is equal to \_\_\_\_\_.
10. Two distinct points are chosen from the given 81 points. Let  $p$  denote the probability that they are friends. Then the value of  $36p$  is equal to \_\_\_\_\_.